

Single Spin Asymmetries in $\ell p^\uparrow \rightarrow h X$ processes and TMD factorisation

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Some estimates for the transverse Single Spin Asymmetry, A_N , in the inclusive processes $\ell p^\uparrow \rightarrow h X$, given in a previous paper, are expanded and compared with new experimental data. The predictions are based on the Sivers distributions and the Collins fragmentation functions which fit the azimuthal asymmetries measured in Semi-Inclusive Deep Inelastic Scattering (SIDIS) processes ($\ell p^\uparrow \rightarrow \ell' h X$). The factorisation in terms of Transverse Momentum Dependent distribution and fragmentation functions (TMD factorisation) – *i.e.* the theoretical framework in which SIDIS azimuthal asymmetries are analysed – is assumed to hold also for the inclusive process $\ell p \rightarrow h X$ at large P_T . The values of A_N thus obtained agree in sign and shape with the data. Some predictions are given for future experiments.

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I. INTRODUCTION

In a previous paper [1] the issue of the validity of the TMD factorisation for hard inclusive processes in which only one large scale is detected has been investigated in a simple phenomenological approach. We considered transverse Single Spin Asymmetries (SSAs) for the $\ell p^\uparrow \rightarrow h X$ process, with the detection, in the lepton-proton center of mass (*c.m.*) frame, of a single large P_T final particle, typically a pion. Also the case of jet production, $\ell p^\uparrow \rightarrow \text{jet} + X$, was considered. The final lepton is not necessarily observed; however, a large value of P_T implies, at leading perturbative order, large values of Q^2 , and the active role of a hard elementary interaction, $\ell q \rightarrow \ell' q$. Such a measurement is the exact analogue of the SSAs observed in the $pp^\uparrow \rightarrow h X$ processes, the well known and large left-right asymmetries A_N , measured over a huge energy range [2–12]. On the other hand, the process is essentially a Semi-Inclusive Deep Inelastic Scattering (SIDIS) process, for which, at large Q^2 values (and small P_T in the $\gamma^* - p$ *c.m.* frame), the TMD factorisation is proven to hold [13–17].

We computed these SSAs assuming the TMD factorisation and using the relevant TMDs (Sivers and Collins functions) as extracted from SIDIS data. A similar idea of computing left-right asymmetries in SIDIS processes, although with different motivations and still demanding the observation of the final lepton, has been discussed in Ref. [18]. A first simplified study of A_N in $\ell p^\uparrow \rightarrow h X$ processes was performed in Ref. [19]. The process was also considered in Refs. [20, 21] in the framework of collinear factorisation with twist-three correlation functions, obtaining asymmetries with a sign opposite to that of the corresponding ones in pp processes. Jet production in $\ell p \rightarrow \text{jet} + X$ was studied in Ref. [22], in a collinear factorisation scheme with a higher-twist quark-gluon-quark correlator, T_F , which is related to the first moment of the Sivers function [23–26].

While at the time of publication of Ref. [1] no data were available on A_N from lepton-proton inclusive processes, very recently some experimental results have been published by the HERMES Collaboration [27]. New data are also available by the JLab Hall A Collaboration [28], but their P_T values are too small (less than 0.7 GeV) to fix a large scale.

We consider here the results of the HERMES Collaboration, selecting those which best fulfil the kinematical conditions necessary for the validity of our scheme, and compare them with our calculations based on TMD factorisation and the Sivers and Collins functions extracted from SIDIS data. In Section II we briefly summarise our formalism and in Section III we compare our numerical results with data and give some predictions for future measurements. Some final comments are given in Section IV.

II. FORMALISM

In Ref. [1] (to which we refer for all details) we considered the process $p^\uparrow \ell \rightarrow h X$ in the proton-lepton *c.m.* frame (with the polarised proton moving along the positive Z_{cm} axis) and the transverse Single Spin Asymmetry:

$$A_N = \frac{d\sigma^\uparrow(\mathbf{P}_T) - d\sigma^\downarrow(\mathbf{P}_T)}{d\sigma^\uparrow(\mathbf{P}_T) + d\sigma^\downarrow(\mathbf{P}_T)} = \frac{d\sigma^\uparrow(\mathbf{P}_T) - d\sigma^\uparrow(-\mathbf{P}_T)}{2 d\sigma^{\text{unp}}(\mathbf{P}_T)}, \quad (1)$$

where

$$d\sigma^{\uparrow,\downarrow} \equiv \frac{E_h d\sigma^{p^{\uparrow,\downarrow} \ell \rightarrow h X}}{d^3 \mathbf{P}_h} \quad (2)$$

is the cross section for the inclusive process $p^{\uparrow,\downarrow} \ell \rightarrow h X$ with a transversely polarised proton with spin “up” (\uparrow) or “down” (\downarrow) with respect to the scattering plane [1]. A_N can be measured either by looking at the production of hadrons at a fixed transverse momentum \mathbf{P}_T , changing the incoming proton polarisation from \uparrow to \downarrow , or keeping a fixed proton polarisation and looking at the hadron production to the left and the right of the Z_{cm} axis (see Fig. 1 of Ref. [1]). A_N was defined (and computed) for a proton in a pure spin state with a pseudo-vector polarisation \mathbf{S}_T normal (N) to the production plane and $|\mathbf{S}_T| = S_T = 1$. For a generic transverse polarisation along an azimuthal direction ϕ_S in the chosen reference frame, in which the \uparrow direction is given by $\phi_S = \pi/2$, and a polarisation $S_T \neq 1$, one has:

$$A(\phi_S, S_T) = \mathbf{S}_T \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{P}}_T) A_N = S_T \sin \phi_S A_N, \quad (3)$$

where \mathbf{p} is the proton momentum. Notice that if one follows the usual definition adopted in SIDIS experiments, one simply has:

$$A_{TU}^{\sin \phi_S} \equiv \frac{2}{S_T} \frac{\int d\phi_S [d\sigma(\phi_S) - d\sigma(\phi_S + \pi)] \sin \phi_S}{\int d\phi_S [d\sigma(\phi_S) + d\sigma(\phi_S + \pi)]} = A_N. \quad (4)$$

Assuming the validity of the TMD factorisation scheme for the process $p \ell \rightarrow h X$ in which the only large scale detected is the transverse momentum P_T of the final hadron in the proton-lepton *c.m.* frame, the main contribution to A_N comes from the Sivers and Collins effects, and one has [1, 29–31]:

$$A_N = \frac{\sum_{q, \{\lambda\}} \int \frac{dx dz}{16 \pi^2 x z^2 s} d^2 \mathbf{k}_\perp d^3 \mathbf{p}_\perp \delta(\mathbf{p}_\perp \cdot \hat{\mathbf{p}}'_q) J(p_\perp) \delta(\hat{s} + \hat{t} + \hat{u}) [\Sigma(\uparrow) - \Sigma(\downarrow)]^{q\ell \rightarrow q\ell}}{\sum_{q, \{\lambda\}} \int \frac{dx dz}{16 \pi^2 x z^2 s} d^2 \mathbf{k}_\perp d^3 \mathbf{p}_\perp \delta(\mathbf{p}_\perp \cdot \hat{\mathbf{p}}'_q) J(p_\perp) \delta(\hat{s} + \hat{t} + \hat{u}) [\Sigma(\uparrow) + \Sigma(\downarrow)]^{q\ell \rightarrow q\ell}}, \quad (5)$$

with

$$\begin{aligned} \sum_{\{\lambda\}} [\Sigma(\uparrow) - \Sigma(\downarrow)]^{q\ell \rightarrow q\ell} &= \frac{1}{2} \Delta^N f_{q/p^\uparrow}(x, k_\perp) \cos \phi \left[|\hat{M}_1^0|^2 + |\hat{M}_2^0|^2 \right] D_{h/q}(z, p_\perp) \\ &+ h_{1q}(x, k_\perp) \hat{M}_1^0 \hat{M}_2^0 \Delta^N D_{h/q^\uparrow}(z, p_\perp) \cos(\phi' + \phi_q^h) \end{aligned} \quad (6)$$

and

$$\sum_{\{\lambda\}} [\Sigma(\uparrow) + \Sigma(\downarrow)]^{q\ell \rightarrow q\ell} = f_{q/p}(x, k_\perp) \left[|\hat{M}_1^0|^2 + |\hat{M}_2^0|^2 \right] D_{h/q}(z, p_\perp). \quad (7)$$

All functions and all kinematical and dynamical variables appearing in the above equations are exactly defined in Ref. [1] and its Appendices and in Ref. [30]. We simply recall here their meaning and physical interpretation.

- \mathbf{k}_\perp is the transverse momentum of the parton in the proton and \mathbf{p}_\perp is the transverse momentum of the final hadron with respect to the direction of the fragmenting parent parton, with momentum \mathbf{p}'_q . ϕ is the azimuthal angle of \mathbf{k}_\perp .

- The first term on the r.h.s. of Eq. (6) shows the contribution to A_N of the Siverson function $\Delta^N f_{q/p\uparrow}(x, k_\perp)$ [32–34],

$$\begin{aligned}\Delta \hat{f}_{q/p,S}(x, \mathbf{k}_\perp) &= \hat{f}_{q/p,S}(x, \mathbf{k}_\perp) - \hat{f}_{q/p,-S}(x, \mathbf{k}_\perp) \equiv \Delta^N f_{q/p\uparrow}(x, k_\perp) \mathbf{S}_T \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp) \\ &= -2 \frac{k_\perp}{M} f_{1T}^{\perp q}(x, k_\perp) \mathbf{S}_T \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp).\end{aligned}\quad (8)$$

It couples to the unpolarised elementary interaction ($\propto (|M_1^0|^2 + |M_2^0|^2)$) and the unpolarised fragmentation function $D_{h/q}(z, p_\perp)$; the $\cos \phi$ factor arises from the $\mathbf{S}_T \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp)$ correlation factor.

- The second term on the r.h.s. of Eq. (6) shows the contribution to A_N of the unintegrated transversity distribution $h_{1q}(x, k_\perp)$ coupled to the Collins function $\Delta^N D_{h/q\uparrow}(z, p_\perp)$ [34, 35],

$$\begin{aligned}\Delta \hat{D}_{h/q\uparrow}(z, \mathbf{p}_\perp) &= \hat{D}_{h/q\uparrow}(z, \mathbf{p}_\perp) - \hat{D}_{h/q\downarrow}(z, \mathbf{p}_\perp) \equiv \Delta^N D_{h/q\uparrow}(z, p_\perp) \mathbf{s}_q \cdot (\hat{\mathbf{p}}'_q \times \hat{\mathbf{p}}_\perp) \\ &= \frac{2p_\perp}{z m_h} H_1^{\perp q}(z, p_\perp) \mathbf{s}_q \cdot (\hat{\mathbf{p}}'_q \times \hat{\mathbf{p}}_\perp).\end{aligned}\quad (9)$$

This non perturbative effect couples to the spin transfer elementary interaction ($d\hat{\sigma}^{q^\uparrow \ell \rightarrow q^\uparrow \ell} - d\hat{\sigma}^{q^\uparrow \ell \rightarrow q^\downarrow \ell} \propto \hat{M}_1^0 \hat{M}_2^0$). The factor $\cos(\phi' + \phi_q^h)$ arises from phases in the \mathbf{k}_\perp -dependent transversity distribution, the Collins function and the elementary polarised interaction.

Some final comments on the kinematical configuration and the notations adopted in the HERMES experiment, with respect to those of Ref. [1], are necessary. According to the usual conventions adopted for SIDIS processes, in HERMES paper [27] the lepton is assumed to move along the positive Z_{cm} axis, so that the processes we are considering here are $\ell p^\uparrow \rightarrow h X$, rather than $p^\uparrow \ell \rightarrow h X$. In this reference frame the \uparrow (\downarrow) direction is still along the $+Y_{cm}$ ($-Y_{cm}$) axis as in Ref. [1] and, keeping the usual definition of $x_F = 2P_L/\sqrt{s}$, where P_L is the longitudinal momentum of the final hadron, only its sign is reversed.

The azimuthal dependent cross section measured by HERMES is defined as [27]:

$$d\sigma = d\sigma_{UU}[1 + S_T A_{UT}^{\sin \psi} \sin \psi], \quad (10)$$

where

$$\sin \psi = \mathbf{S}_T \cdot (\hat{\mathbf{P}}_T \times \hat{\mathbf{k}}) \quad (11)$$

coincides with our $\sin \phi_S$ of Eq. (3), as \mathbf{p} and \mathbf{k} (respectively, the proton and the lepton 3-momenta) are opposite vectors in the lepton-proton *c.m.* frame. Indeed, taking into account that “left” and “right” are interchanged in Refs. [1] and [27] (as these are defined looking downstream along opposite directions, respectively the proton and the lepton momentum directions) and the definition of x_F , one has:

$$A_{UT}^{\sin \psi}(x_F, P_T) = A_N^{p^\uparrow \ell \rightarrow hX}(-x_F, P_T), \quad (12)$$

where $A_N^{p^\uparrow \ell \rightarrow hX}$ is the SSA as given by Eq. (5) and computed in Ref. [1], and $A_{UT}^{\sin \psi}$ is the quantity measured by HERMES [27].

III. ESTIMATES FOR $A_{UT}^{\sin \psi}$, COMPARISONS WITH DATA AND PREDICTIONS

In this Section we present our estimates for $A_{UT}^{\sin \psi}$, following the notation and convention adopted by the HERMES experiment. In our computation, based on the TMD factorisation, we consider two different sets of Siverson and Collins functions (the latter coupled to the transversity distribution), as previously obtained in a series of papers from fits of SIDIS and e^+e^- data [36–39].

These sets, besides some different initial assumptions, differ in the choice of the collinear fragmentation functions (FFs). More precisely, we adopt the Siverson functions extracted in Ref. [36], where only up and down quark contributions were considered, together with the first extraction of the transversity and Collins functions obtained in Ref. [37]. In such studies we adopted, and keep using here, the Kretzer set for the collinear FFs [40]. We shall refer to this set of functions as the SIDIS 1 set.

We then consider a more recent extraction of the Siverson functions [38], where also the sea quark contributions were included, together with an updated version of the transversity and Collins functions [39]; in these cases we adopted

another set for the FFs, namely that one by de Florian, Sassot and Stratmann (DSS) [41]. We shall refer to these as the SIDIS 2 set.

The use of these two sets of parameterisations, with their peculiar differences, allows to take into account both the role of different weights between leading and non-leading collinear FFs, as well as the different behaviour in the large x region of the Sivers and transversity distributions. The large x behaviour of these functions is still largely unconstrained by SIDIS data, while it might be relevant to explain the values of A_N measured in $p^\uparrow p \rightarrow \pi X$ processes at RHIC, as studied in Refs. [42, 43]. As this paper focuses on a process kinematically much closer to SIDIS, the large x behaviour of the involved TMDs is not so relevant here. Our two sets of TMDs (SIDIS 1 and SIDIS 2) are well representative of the possible uncertainties.

We then simply compute the values of A_N as resulting, in the TMD factorised scheme, from the – *SIDIS and e^+e^- extracted* – SIDIS 1 and SIDIS 2 sets of TMDs. We will also show the uncertainty bands obtained by combining the statistical uncertainty bands of the Sivers and Collins functions, given by the procedure described in Appendix A of Ref. [38].

In the following we will consider both the fully inclusive data from $\ell p \rightarrow \pi X$ processes at large P_T , as well as the sub-sample of data from processes in which also the final lepton is tagged (SIDIS category). In the first case there is only one large scale, the P_T of the final pion, and for $P_T \simeq 1$ GeV, in order to avoid the low Q^2 region, one has to look at pion production in the backward proton hemisphere (according to the HERMES conventions this means $x_F > 0$). In this region (large P_T and $x_F > 0$) the lepton-quark scattering is still dominated by $Q^2 \geq 1$ GeV² and our pQCD computation is under control.

For the tagged-lepton sub-sample data Q^2 is measured and chosen to be always bigger than 1 GeV². Notice that even in such conditions, working in the lepton-proton *c.m.* frame, P_T is still defined as the transverse momentum of the pion w.r.t. the lepton-proton direction. We will refer to these data as “SIDIS category”.

Another important aspect to keep in mind is that in both cases (inclusive or lepton-tagged events) one is not able to separate the single contributions to A_N of the Sivers and Collins effects, that in principle could contribute together.

A. Fully inclusive case

In this case, in order to apply our TMD factorised approach, one has to consider data at large P_T . Among the HERMES data there is one bin that fulfils this requirement, with $1 \lesssim P_T \lesssim 2.2$ GeV, and $\langle P_T \rangle \simeq 1\text{--}1.1$ GeV. In Figs. 1 and 2 we show a comparison of our estimates with these data, respectively for positive and negative pion production. More precisely, we show the results coming from both sets of TMDs, SIDIS 1 (left panels) and SIDIS 2 (right panels), for the Sivers (dotted blue lines) and Collins (dashed green lines) effects separately, together with their sum (solid red lines). We also computed the statistical uncertainty bands for both effects and showed the envelope of the two bands (shaded area). Some comments are in order:

- In this kinematical region the Collins effect is always negligible, almost compatible with zero. The reason is twofold: from one side the partonic spin transfer in the backward proton hemisphere is dynamically suppressed, as explained in Ref. [1]; secondly, the azimuthal phase (see the second term on the r.h.s. of Eq. (6)) oscillates strongly, washing out the effect.
- The Sivers effect does not suffer from any dynamical suppression, since it enters with the unpolarised partonic cross section. Moreover, there is no suppression from the integration over the azimuthal phases, as it happens, for instance, in $pp \rightarrow \pi X$ case. Indeed in $\ell p \rightarrow \pi X$ only one partonic channel is at work and, for the moderate Q^2 values of HERMES kinematics, the Sivers phase (ϕ) appearing in the first term on the r.h.s. of Eq. (6) appears also significantly in the elementary interaction, thus resulting in a non-zero phase integration.
- Moreover, in this kinematical region, even if looking at the backward hemisphere of the polarised proton, one probes its valence region, where the extracted Sivers function are well constrained. The reason is basically related to the moderate *c.m.* energy, $\sqrt{s} \simeq 7$ GeV, of the HERMES experiment.
- The difference between SIDIS 1 and SIDIS 2 results for the negative pion case, Fig. 2, comes from the fact that in the first case the Sivers function for up quark plays a relative bigger role, even if coupled with the non-leading FF.
- The results presented here for the SIDIS 2 set of TMDs correspond to the predictions given in Ref. [1], with the difference that they were obtained for $P_T = 1.5$ and 2.5 GeV, and one should change x_F into $-x_F$.

As one can see, while the SSA for positive pion production is a bit overestimated, Fig. 1, the description of the negative pion SSAs is in fair agreement with data for the SIDIS 1 set (left panel in Fig. 2). Notice that in the fully

inclusive case under study, at such values of \sqrt{s} and Q^2 other effects could contaminate the SSA. Nonetheless the qualitative description, in size, shape and sign, is quite encouraging.

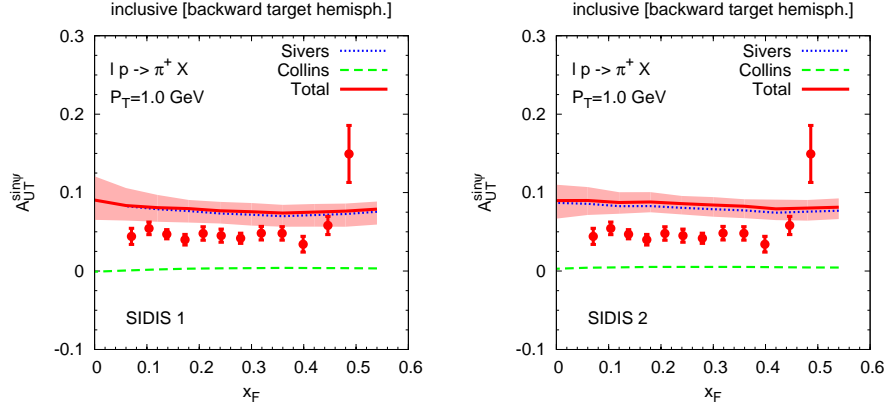


FIG. 1: The theoretical estimates for $A_{UT}^{\sin\psi}$ vs. x_F at $\sqrt{s} \simeq 7$ GeV and $P_T = 1$ GeV for inclusive π^+ production in $\ell p^\uparrow \rightarrow \pi X$ processes, computed according to Eqs. (12) and (5)–(7) of the text, are compared with the HERMES data [27]. The contributions from the Sivers (dotted blue lines) and the Collins (dashed green lines) effects are shown separately and also added together (solid red lines). The computation is performed adopting the Sivers and Collins functions of Refs. [36, 37], referred to as SIDIS 1 in the text (left panel), and of Refs. [38, 39], SIDIS 2 in the text (right panel). The overall statistical uncertainty band, also shown, is the envelope of the two independent statistical uncertainty bands obtained following the procedure described in Appendix A of Ref. [38].

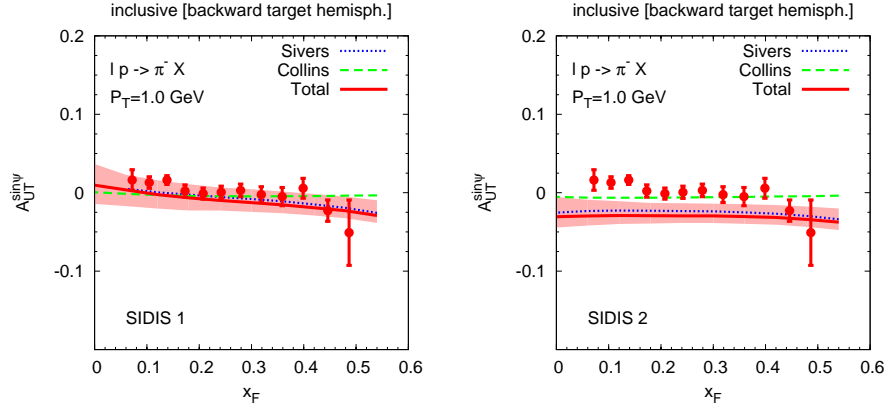


FIG. 2: Same as in Figure 1 but for inclusive π^- production.

B. Tagged or semi-inclusive category

The HERMES Collaboration presents also a sub-sample of ℓp data where the final lepton is tagged [27]. Of course the number of these events is strongly reduced w.r.t. the fully inclusive case. Nonetheless the observed asymmetries are sizeable and show a peculiar behaviour.

We then consider also these data by imposing HERMES cuts: $Q^2 > 1$ GeV², $W^2 > 10$ GeV², $0.023 < x_B < 0.4$, $0.1 < y < 0.95$ and $0.2 < z_h < 0.7$, where these are the standard variables adopted for the study of SIDIS processes. Even in this case we restrict the analysis to the large P_T region, namely $P_T > 1$ GeV. In fact, in contrast to the SIDIS azimuthal asymmetries analysed in the $\gamma^* - p$ *c.m.* frame, where the low $P_T \leq 1$ GeV of the final hadron is entirely given at leading order in terms of the intrinsic transverse momenta in the distribution and fragmentation functions, here, working in the $\ell - p$ *c.m.* frame, the observed P_T is also given by the hard scattering process. For this reason, to be sensitive to the intrinsic transverse momentum effects, one has not to consider necessarily very small P_T values.

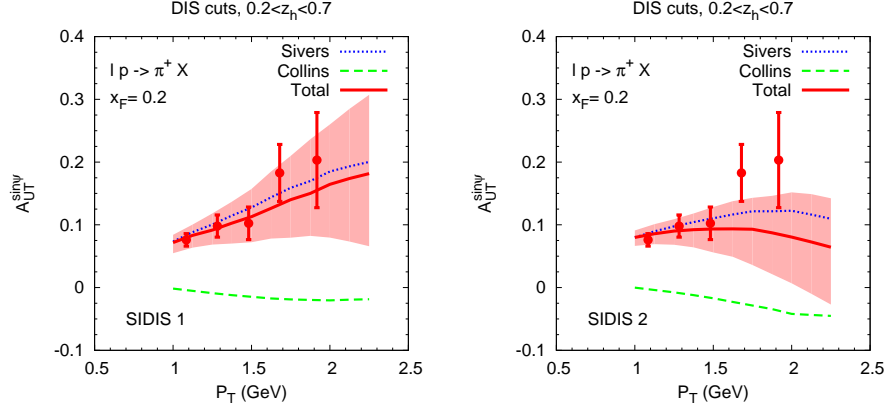


FIG. 3: The theoretical estimates for $A_{UT}^{\sin \psi}$ vs. P_T at $\sqrt{s} \simeq 7$ GeV and $x_F = 0.2$ for inclusive π^+ production for the lepton tagged events in $\ell p^\dagger \rightarrow \pi X$ process, computed according to Eqs. (12) and (5)–(7) of the text, are compared with the HERMES data [27]. The contributions from the Sivers (dotted blue lines) and the Collins (dashed green lines) effects are shown separately and also added together (solid red lines). The computation is performed adopting the Sivers and Collins functions of Refs. [36, 37], referred as SIDIS 1 in the text (left panel), and of Refs. [38, 39], SIDIS 2 in the text (right panel). The overall statistical uncertainty band, also shown, is the envelope of the two independent statistical uncertainty bands obtained following the procedure described in Appendix A of Ref. [38].

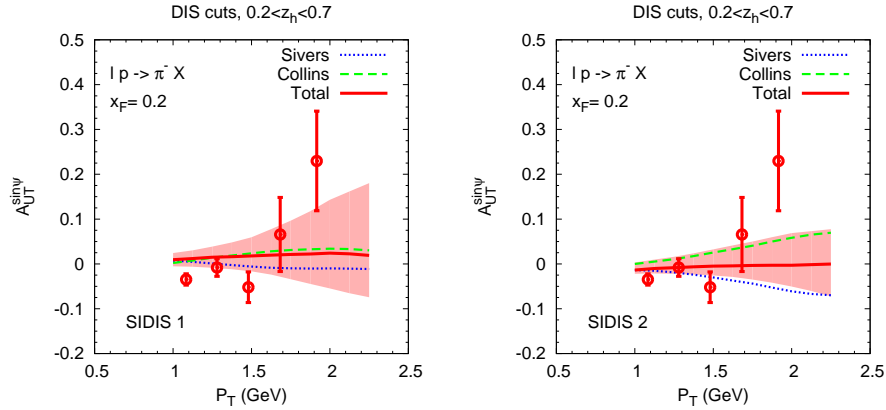


FIG. 4: Same as in Figure 3 but for π^- production.

We show our estimates compared with HERMES data in Figs. 3 and 4, respectively for positive and negative pion production as a function of P_T at fixed $x_F = 0.2$. Notice that for $P_T > 1$ GeV, the values of x_F probed in the HERMES kinematics are all very close to 0.2. We checked that increasing the value of x_F , up to 0.3, the results are almost unchanged. Again, we show the contributions from the Sivers (dotted blue line) and Collins (dashed green line) effects separately and added together (solid red line) with the overall uncertainty bands (shaded area). Some comments follow:

- In this region the Collins effect (dashed green lines) is only partially suppressed by the dynamics and the azimuthal phase integration. Indeed the spin transfer is still sizeable and the azimuthal phase entering the Collins effect is peaked around π , that is the $\cos(\phi' + \phi_q^h)$ in the second term on the r.h.s. of Eq. (6) is peaked around -1 . Keeping in mind that the partonic spin transfer is always positive and that the convolution of the transversity distributions with the Collins functions is positive for π^+ and negative for π^- , one can understand the sign of this contribution. The difference between the SIDIS 1 and the SIDIS 2 sets (a factor around 2-3) comes from the different behaviour of the quark transversity functions at moderately large x .
- The Sivers effect (dotted blue lines) for π^+ production (Fig. 3) is sizeable for both sets. On the other hand for π^- production (Fig. 4) the SIDIS 1 set (left panel) gives almost zero due to the strong cancellation between the unsuppressed Sivers up quark distribution coupled to the non-leading FF, with the more suppressed down

quark distribution. In the SIDIS 2 set (right panel), the same large x behaviour of the up and down quark Sivers distributions implies no cancellation.

- With the exception of the largest P_T data point the description of the data in terms of the sum of these effects is fairly good for both sets.

C. Predictions

Data at $P_T \simeq 1$ GeV are expected from the future JLab 12 operation at 11 GeV. Because of the rather low $c.m.$ energy ($\sqrt{s} \simeq 4.8$ GeV), in order to select data with large values of Q^2 one has to consider a backward (w.r.t. the proton direction) production, which means $x_F \geq 0.1$. With these kinematical bounds most contribution come from the quark valence region. Our predictions, analogous to the results presented in Figs. 1 and 2, are shown in Figs. 5 and 6. The results expected at JLab 12 are similar to those observed at HERMES.

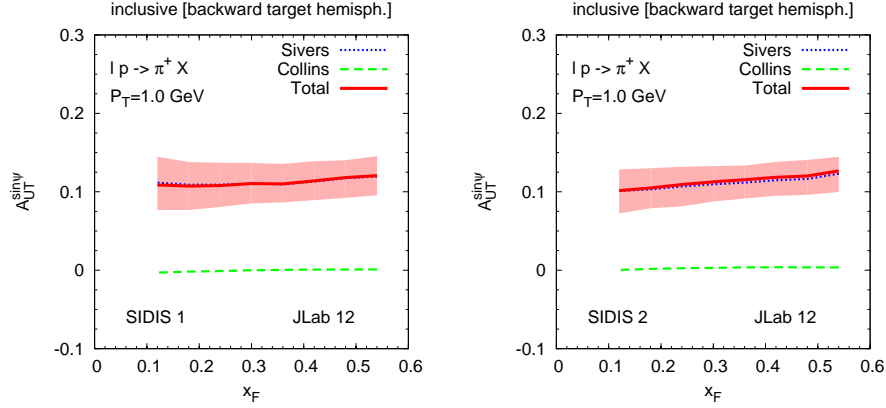


FIG. 5: Theoretical predictions for $A_{UT}^{\sin \psi}$ vs. x_F at $\sqrt{s} \simeq 4.8$ GeV and $P_T = 1$ GeV for inclusive π^+ production in $\ell p^\uparrow \rightarrow \pi X$ processes, computed according to Eqs. (12) and (5)–(7) of the text, are shown for future JLab experiments. The contributions from the Sivers (dotted blue lines) and the Collins (dashed green lines) effects are shown separately and also added together (solid red lines). The computation is performed adopting the Sivers and Collins functions of Refs. [36, 37], referred to as SIDIS 1 in the text (left panel), and of Refs. [38, 39], SIDIS 2 in the text (right panel). The overall statistical uncertainty band, also shown, is the envelope of the two independent statistical uncertainty bands obtained following the procedure described in Appendix A of Ref. [38].

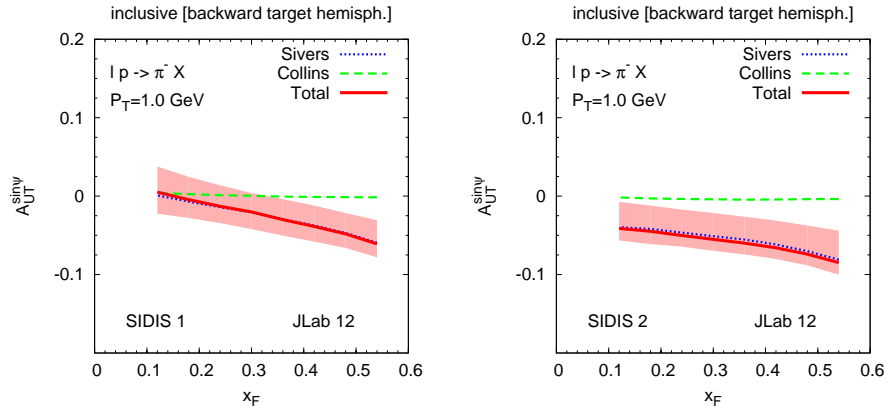


FIG. 6: Same as in Figure 5 but for inclusive π^- production.

One might also wonder whether some features that characterise the SSAs observed in $pp \rightarrow \pi X$ processes and that can be reproduced within a TMD factorisation scheme [42], could still be encountered in $\ell p \rightarrow \pi X$ reactions. To

answer this question we consider the inclusive ℓp process at $\sqrt{s} = 50$ GeV. In this case, in order to have a more direct comparison with the $p^\dagger p$ case, we calculate A_N as defined in Eq. (6), with the polarised proton moving along Z_{cm} , that is with positive x_F in the forward hemisphere of the polarised proton. In Fig. 7 we show our estimates of A_N for π^0 production in the process $p^\dagger \ell \rightarrow \pi X$ at $\sqrt{s} = 50$ GeV and $P_T = 1$ GeV (left panel) and $P_T = 2$ GeV (right panel) adopting the SIDIS 1 set. This set indeed is the one that better reproduces the behaviour of A_N in $p^\dagger p \rightarrow \pi X$ processes (see for instance Ref. [44]). The result deserves a few comments.

- The Collins effect in the backward region is totally negligible: this is due to a strong suppression coming from the azimuthal phase integration. In the forward region the SIDIS 1 set, as well the SIDIS 2 (results not shown), give tiny values even if the azimuthal phase would be effective. In particular for $x_F > 0.3$, once again the cosine factor entering this effect in Eq. (6) is negative.
- The Sivers effect is sizeable and increasing with x_F for positive values of x_F , while negligible in the negative x_F region. Notice that the suppression of the Sivers effect for $x_F < 0$, even if in such a process there is only one partonic channel, is due to a weak dependence on the azimuthal phase of the elementary interaction at the large Q^2 values reached at this energies.
- It is worth noticing that the functional shape of $A_N(x_F)$, for the $p^\dagger \ell \rightarrow h X$ large P_T process, is similar to that observed at various energies in $p^\dagger p \rightarrow h X$ processes, being negligible at negative x_F and increasing with positive values of x_F .
- The process $\ell p^\dagger \rightarrow \text{jet} + X$ at large \sqrt{s} values was studied in a twist-3 formalism in Ref. [22]. The quark-gluon-quark Qiu-Sterman correlator T_F was fixed exploiting its relation with the Sivers function, taken from an extraction [38] from SIDIS data. The value of A_N was found to be positive for $x_F > 0$ (the same kinematical configuration as for our Fig. 7 was adopted), with results very close to the results we find here for π^0 production and we found in Ref. [1] for jet production. Indeed the twist-3 and the TMD mechanisms were shown to be closely related and provide a unified picture for SSAs in SIDIS processes [45]. However, the factorised twist-3 collinear scheme, using the SIDIS extracted Sivers functions for fixing the Qiu-Sterman correlator T_F , seems to have severe problems in explaining the SSA A_N observed in pp processes, leading to values of A_N opposite to those measured [46]. These issues were further studied in Refs. [47–49]. A recent analysis of A_N in pp scattering in the twist-3 formalism [50] attempts at solving this problem showing that the asymmetry might be dominated by new large effects coming from fragmentation. It is not clear how much these same effects would change the value of A_N in SIDIS when going from jet to π^0 production.
- The measurement of asymmetries in the same kinematical region and with the same features as in Fig. 7 for π^0 production, and as in Fig. 6 of Ref. [1] for jet production, would be a strong indication in support of our TMD factorised approach. Such measurements might be possible at a future Electron-Ion-Collider (EIC) [51].

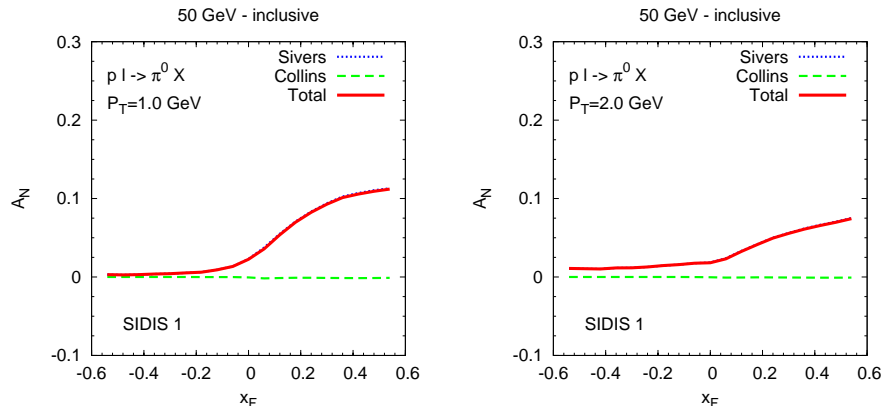


FIG. 7: Theoretical estimates of A_N vs. x_F at $\sqrt{s} \simeq 50$ GeV, $P_T = 1$ GeV (left panel) and $P_T = 2$ GeV (right panel) for inclusive π^0 production in the $p^\dagger \ell \rightarrow \pi X$ process. Notice that, contrary to the kinematical configurations of Figs. 1 and 2, a forward production w.r.t. the proton direction corresponds here to positive values of x_F . The contributions from the Sivers (dotted blue lines) and the Collins (dashed green lines) effects are shown separately and also added together (solid red lines). The estimates are obtained adopting the Sivers and Collins functions of Refs. [36, 37] (SIDIS 1 set), according to Eqs. (5)–(7) of the text.

IV. COMMENTS AND CONCLUSIONS

We have further pursued and tested the idea presented in Ref. [1] for assessing the validity of the TMD factorisation in inclusive processes in which a single large P_T particle is produced. Starting from the TMD factorisation valid for SIDIS processes, $\ell p \rightarrow \ell h X$, in which a large Q^2 virtual photon γ^* hits a quark, which then fragments into a final hadron with a small P_T in the $\gamma^* - p$ *c.m.* frame, we have assumed its validity for processes in which the final lepton is not necessarily observed, but the final detected hadron has a large P_T in the lepton-proton *c.m.* frame. A large value of P_T implies, at leading order, a large angle elementary scattering, $\ell q \rightarrow \ell q$, and then a large value of Q^2 . Such a process is analogous to the $p^\dagger p \rightarrow h X$ processes, for which large SSAs A_N , Eq. (1), have been measured. According to the TMD factorisation approach, the SSAs can be generated by the Sivers and Collins effects [42, 43].

We have computed the single spin asymmetry A_N , for the $\ell p^\dagger \rightarrow h X$ process and in the TMD factorised scheme, as generated by the Sivers and the Collins functions, which have been extracted from SIDIS and e^+e^- data [36–39]. Doing so, we adopt a unified TMD factorised approach, valid for $\ell p \rightarrow \ell h X$ and $\ell p \rightarrow h X$ processes, in which, consistently, we obtain information on the TMDs and make predictions for A_N . Some of these predictions were given in Ref. [1].

New HERMES data on A_N are now available [27] for different kinematical regions; we have selected those data which – although not yet optimally – fulfil the conditions of applicability of our TMD factorisation approach, and compared them with the results of our computations. We have selected two sets of TMDs extracted from SIDIS and e^+e^- data, and which are representative, with their large differences, of the uncertainties which the SIDIS available data still allow.

It turns out, Figs. 1–4, that our theoretical estimates for $A_{UT}^{\sin\psi}(x_F, P_T) = A_N(-x_F, P_T)$ agree well, in shape and sign, with the experimental results, in particular for one set of TMDs (SIDIS 1). In some cases (see Fig. 1) our results are a bit larger than data, yet with the right sign and behaviour; one should not forget that in the kinematical regions we are considering (in P_T , Q^2 and \sqrt{s}) other mechanisms might still be at work. The overall agreement between our computations and the data is very encouraging.

In Figs. 5 and 6 we have estimated the expected value of $A_{UT}^{\sin\psi}$ at the future JLab experiments at 12 GeV. We are still in a kinematical region where a careful selection of data is necessary in order to ensure the validity of our approach. The results are similar to those obtained in Figs. 1 and 2 for HERMES kinematics.

At last, in Fig. 7, we have given predictions for $A_N(x_F)$ in very safe kinematical regions for our approach to hold. Indeed, as expected, we recover the same behaviour for $A_N(x_F)$ as observed in $p^\dagger p \rightarrow \pi^0 X$ processes. Such a prediction, crucial for assessing the validity of our TMD factorisation scheme, could be tested at a future EIC [51].

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